Student Workbook

Sequences and Their Notations

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How can we describe and predict patterns in ordered lists of numbers using explicit formulas, recursive formulas, and sigma notation?

Lesson Overview

A sequence is an ordered list of numbers called terms, written $a_1, a_2, a_3, \ldots, a_n$, where the subscript gives the term's position. A sequence can be defined explicitly (a formula that gives any term directly from its position) or recursively (each term built from the one before it, plus a starting value). Sigma (Σ) notation gives a compact way to write sums of sequence terms.

$$a_n = f(n) \text{ (explicit)}$$
$$a_n = f(a_{n-1}) \text{ (recursive)}$$
$$\Sigma(k=m \text{ to } n)\ f(k)$$

Explicit Formula	$a_n = f(n)$ — plug in any position n and get the term directly.
Recursive Formula	$a_n = f(a_{n-1})$ — each term depends on the previous one; requires a starting value a_1.
Sigma Notation	Σ(k=1 to n) f(k) means add f(k) for every integer k from 1 to n.
Finite vs. Infinite	A sequence is finite if it has a last term, infinite if it continues without end (shown by '...').

Worked Examples

EXAMPLE 1

Write the first 5 terms of the sequence defined by $a_n = 3n - 1$.

- Substitute n = 1: $a_1 = 3(1) - 1 = 2$
- Substitute n = 2: $a_2 = 3(2) - 1 = 5$; n = 3: $a_3 = 8$; n = 4: $a_4 = 11$; n = 5: $a_5 = 14$

Answer: The first 5 terms are 2, 5, 8, 11, 14.

EXAMPLE 2

Write the first 5 terms of the recursive sequence: $a_1 = 2$, $a_n = a_{n-1} + 4$.

- Start with $a_1 = 2$ (given).
- $a_2 = a_1+4 = 6$; $a_3 = a_2+4 = 10$; $a_4 = a_3+4 = 14$; $a_5 = a_4+4 = 18$

Answer: The first 5 terms are 2, 6, 10, 14, 18.

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EXAMPLE 3

Find an explicit formula for the sequence 5, 8, 11, 14, 17.

- Find the common difference: $8 - 5 = 3$, so $d = 3$.
- The formula has the form $a_n = 3n + c$. Use $n = 1$: $3(1) + c = 5 \rightarrow c = 2$.
- Check $n = 2$: $3(2)+2 = 8$ ✓; $n = 3$: $3(3)+2 = 11$ ✓

Answer: $a_n = 3n + 2$

EXAMPLE 4

Evaluate Σ(k=1 to 4) of (2k + 1).

- k=1: 2(1)+1=3; k=2: 2(2)+1=5; k=3: 2(3)+1=7; k=4: 2(4)+1=9
- Sum: $3 + 5 + 7 + 9 = 24$

Answer: Σ(k=1 to 4)(2k + 1) = 24

EXAMPLE 5

Consider the sequence 1, 1/2, 1/4, 1/8, … Is it finite or infinite? Write sigma notation for the sum of the first 4 terms.

- The '...' indicates the sequence continues without end — it is infinite.
- The general term is $a_n = (1/2)^{n-1}$.
- Sigma notation for the first 4 terms: Σ(k=1 to 4) $(1/2)^{k-1}$.

Answer: Infinite. Σ(k=1 to 4) $(1/2)^{k-1}$ = 1 + 1/2 + 1/4 + 1/8 = 15/8.

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Write the first 5 terms of the sequence defined by $a_n = n^2 - 2$.

Hint: Substitute n = 1, 2, 3, 4, 5 into the formula one at a time.

2 Write the first 4 terms of the recursive sequence: $a_1 = 3$, $a_n = 2a_{n-1} - 1$.

Hint: Start with $a_1 = 3$, then apply the rule $a_n = 2a_{n-1} - 1$ repeatedly.

3 Find an explicit formula for the sequence 4, 7, 10, 13, 16.

Hint: Find the common difference first, then use a_1 to solve for the constant.

4 Evaluate $\Sigma(k=1 \text{ to } 5)\ k^2$.

Hint: Write out k^2 for k = 1, 2, 3, 4, 5 and add the results.

5 Write sigma notation for the sum 3 + 6 + 9 + 12 + 15.

Hint: Each term is a multiple of 3. Express the kth term as 3k and identify the bounds.

Key Vocabulary

Sequence	An ordered list of numbers, each called a term.
Explicit formula	A formula $a_n = f(n)$ that gives any term directly from its position n.
Recursive formula	A formula that defines each term using one or more previous terms, plus an initial condition.
Sigma notation	Compact notation $\Sigma(k=m \text{ to } n)\ f(k)$ representing the sum of f(k) over consecutive integers.
Finite sequence	A sequence with a definite last term.
Infinite sequence	A sequence that continues without end, indicated by '...'.

Quick Check Quiz

Circle the letter of the correct answer for each question.

1 **Which of the following is an explicit formula?** Multiple Choice

A $a_n = a_{n-1} + 3$
B $a_n = 4n - 1$
C $a_n = 2a_{n-1}$
D $a_1 = 5,\ a_n = a_{n-1} - 2$

2 **What is the 4th term of $a_n = 2n + 3$?** Multiple Choice

A 9
B 11
C 13
D 7

3 **For the recursive sequence $a_1 = 1$, $a_n = a_{n-1} + 5$, what is a_3?** Multiple Choice

A 6
B 11
C 16
D 10

4 **Evaluate Σ(k=1 to 3) (k + 2).** Multiple Choice

- **A** 12
- **B** 15
- **C** 9
- **D** 18

5 **Which sequence is infinite?** Multiple Choice

- **A** 1, 3, 5, 7, 9
- **B** The first 10 multiples of 4
- **C** 2, 4, 6, 8, ...
- **D** $a_1 = 1, a_2 = 2, a_3 = 3$

Common Mistakes to Avoid

✗ Using $a_n = a_{n-1} + d$ without a starting value for a recursive formula.

✓ Always state the initial term (e.g., $a_1 = 2$) when writing a recursive formula.

✗ Confusing the index variable in sigma notation with the sequence subscript.

✓ The index k in Σ is a dummy variable — it only lives inside the sum.

✗ Writing $a_n = 3n$ when the first term is not 3 (e.g., sequence starts at 5).

✓ Check your formula against a_1 and adjust the constant: $a_n = 3n + c$.

✗ Assuming every sequence with '...' is infinite.

✓ Context matters — '...' can mean 'continuing to a stated last term' (finite) or 'forever' (infinite).

Math Tips

★ To find an explicit formula for an arithmetic sequence, use $a_n = a_1 + (n - 1)d$.

★ Sigma notation Σ(k=m to n) f(k) means sum f(k) for every integer k from m to n inclusive.

★ A recursive formula always needs two pieces: the rule AND the initial condition.

★ When evaluating a sigma sum, list every term — it prevents arithmetic errors.